Bergen County Mathematics League

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Brief Contest Solutions #5 **2015-2016**

5-1. We have (5 Leus)$\left(\frac{9\text{ Cons}}{3\text{ Leus}}\right)\left(\frac{4\text{ Flegs}}{2\text{ Cons}}\right) = \boxed{30}$ Flegs.

5-2. If N is the least such positive integer, then $N + 2$ leaves a remainder of 0 when divided by 3, 4, or 5. Therefore, $N + 2$ is a multiple of 3, 4, and 5. The least such positive multiple is 60, so the least possible value of $N + 2$ is 60, and $N = \boxed{58}$.

5-3. The sum of the coefficients of any polynomial is equal to the value of the polynomial when $x = 1$. This value is $(1-1)(1-2)(1-3)\times\ldots\times(1-2015)(1-2016) = \boxed{0}$.

5-4. If we solve $f(t) = 3$ algebraically, then we get $t = -3$ or $t = 3$. Thus when $f(f(x)) = 3$, $f(x)$ is either -3 or 3. Graphically, when $f(x) = -3$, we have 2 solutions (because the graph intersects $y = -3$ at two points); when $f(x) = 3$, we have 2 additional solutions, for a total of $\boxed{4}$ solutions.

5-5. Using the property of reference angles, we have $\sum_{i=1}^{12} \sin^2\left(\frac{i\pi}{6}\right) = 2\sin^2(0) + 4\sin^2\left(\frac{\pi}{6}\right) + 4\sin^2\left(\frac{2\pi}{6}\right) + 2\sin^2\left(\frac{3\pi}{6}\right) = 0 + 1 + 3 + 2 = 6$. The sine function cycles periodically so $\sum_{i=1}^{12} \sin^2\left(\frac{i\pi}{6}\right) = \sum_{i=13}^{24} \sin^2\left(\frac{i\pi}{6}\right) = \sum_{i=25}^{36} \sin^2\left(\frac{i\pi}{6}\right) = \ldots = 6$. We get $2016/6 = 336$. We have 336 summations, each with 12 terms, so one value of n is $336 \times 12 = 4032$. Also, since $\sin^2\left(\frac{4032\pi}{6}\right) = 0$, we can disregard the last term and have $n = 4031$. The two values of n are $\boxed{4031, 4032}$.

5-6. In the trapezoid, drop altitudes $\overline{AE}$ and $\overline{BF}$. Let $AE = BF = h$, and let $DE = x$, all as shown. It follows that $FC = 14 - x$. From the Pythagorean Theorem we get both $x^2 + h^2 = 13^2$ and $(14-x)^2 + h^2 = 15^2$. Subtracting one equation from the other, we get $x = 5$ and $h = 12$. The area of the trapezoid is $\frac{1}{2}(10 + 24)\times 12 = \boxed{204}$.