Bergen County Mathematics League

Problem Author:
Jerry C. Leung
www.mathleague.com

Problem Editors:
Steve Conrad & Dan Flegler
www.mathleague.com

Brief Contest Solutions #1

2015-2016

1-1. Putting all nonzero terms on the left and factoring $[(x-2015)-(x-2017)](x-2016) = 0$. This simplifies to $2(x-2016)=0$, so $x = \boxed{2016}$.

1-2. Let $y_1 = 2x+8$, $y_2 = -x+32$, and $y_3 = -20$. Equating, y_1 and y_2 intersect at $A(8, 24)$, y_1 and y_3 intersect at $B(-14, -20)$, and y_2 and y_3 intersect at point $C(52, -20)$. Using $\overline{BC}$ as the base and the segment from A to $\overline{BC}$ as the height, we see that the lengths of the base and height are 66 and 44 respectively. The area is $\frac{1}{2}bh = \frac{1}{2}(66)(44) = \boxed{1452}$.

1-3. Let $m\angle A = x$. Since $\angle A$ and $\angle B$ are complementary, $m\angle B = 90 - x$. Since $\angle B$ and $\angle C$ are supplementary, $m\angle C = 180 - m\angle B = 90 + x$. Since $m\angle C + m\angle D = 360$, $m\angle D = 360 - m\angle C = 270 - x$. Finally, $m\angle D = 8m\angle A$, so $270 - x = 8x$, and $x = \boxed{30 \text{ or } 30^\circ}$.

1-4. Divide the problem into cases:
If the units digit is 0, then the tens digit must be 0 as well, and the hundreds digit can be any of the digits from 1 to 9, for a total of 9 numbers.
If the units digit is a prime (2, 3, 5, or 7), then the tens digit and the hundreds digit must be that prime and 1 (not necessarily in that order), for a total of 8 numbers.
If the units digit is 1, then the only possible number is 111, for just 1 number.
If the units digit is 4, then we have 144, 414, and 224, for a total of 3 numbers.
If the units digit is 6, then we have 166, 616, 236, and 326, for a total of 4 numbers.
If the units digit is 8, then we have 188, 818, 248, and 428, for a total of 4 numbers.
If the units digit is 9, then we have 199, 919, and 339, for a total of 3 numbers.
Altogether, the total of these is $9+8+1+3+4+4+3 = \boxed{32}$.

1-5. The sequence is 20, 4, 16, 37, 58, 89, 145, 42, 20, 4, 16 . . . . It has a repeating pattern for every 8 terms. Since the remainder of 2015/8 is 7, the 2015th term is equal to the 7th term, which is $\boxed{145}$.

1-6. Let O be the center of the circle tangent to $\overline{CD}$, $\widehat{EF}$, and $\widehat{BD}$. Draw a perpendicular from O to $\overline{BC}$ intersecting $\overline{BC}$ at G. If the length of a radius of circle O is r, then $BO = 6 + r$, $BG = 12 - r$, and $OC = 12 - r$. By using the Pythagorean Thm. in $\triangle BOG$ and $\triangle COG$, we get $OG^2 = (6+r)^2 - (12-r)^2$ and $OG^2 = (12-r)^2 - r^2$. Equating and solving, we get $r = \boxed{4.2}$.