

# Bergen County Mathematics League

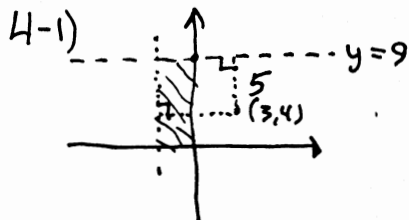
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**Brief Contest Solutions #4**

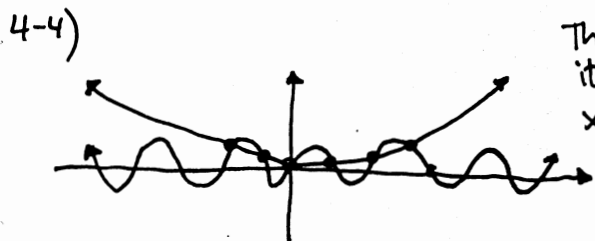
**2014-2015**



Rotate  $y=9$   $90^\circ$  counter clockwise with center  $(3,4)$  and the line that was 5 units above  $(3,4)$  is now 5 units to the left of  $(3,4)$ . It's the line  $x=-2$ . The area sought is  $2 \times 9 = \boxed{18}$ .

4-2) Since  $(1+x)^k = 1 + kx + \frac{k(k-1)}{2}(x^2) + \dots$  For  $x=k, (1+k)^k = 1 + k^2 + \frac{k^3(k-1)}{2} + \dots$   
 $\therefore k^2 = \frac{1}{4}$ , so  $k = \frac{1}{2}$  or  $k = -\frac{1}{2}$ . For  $k = \frac{1}{2}$ ,  $\frac{k^3(k-1)}{2} < 0$   $\therefore = 1 + \frac{1}{4} + \frac{1 \cdot 3}{4 \cdot 8} + \dots$   
 $\therefore k = -\frac{1}{2}$  and  $N = (1 - \frac{1}{2})^{-1/2} = (\frac{1}{2})^{-1/2} = 2^{1/2} = \sqrt{2} \therefore (k, n) = \boxed{(-\frac{1}{2}, \sqrt{2})}$

4-3) If Ty wore a blue suit, he wore either blue socks or a white shirt. But if he wore blue socks, he also wore a white shirt. Thus, blue suit  $\Rightarrow$  white shirt. But blue suit and white shirt  $\Rightarrow$  blue tie. Since Ty always wears a red tie, he did not wear his blue suit. But gray suit  $\Rightarrow$  blue socks, and blue socks  $\Rightarrow$  white shirt. Therefore, Ty's wardrobe for his dates with Miss Fortune is gray suit, white shirt, blue socks.



The period of  $y = \sin 8x$  is  $\frac{\pi}{8}$ , so  $\sin 8x$  reaches its max at  $x = \frac{\pi}{16} + \frac{n\pi}{8}, n \in \mathbb{Z}$ . The parabola  $y = x^2 > 1$  when  $|x| > 1$ . The sine wave will cross the parabola until  $\frac{\pi}{16} + \frac{n\pi}{8} > 1$ , which happens if  $|n| \geq 2$ . As shown, there are  $\boxed{6}$  intersections.

4-5) For any 5 tickets, there are  $5! = 120$  ways to order them, only one of which is in increasing order. Thus,  $P = \boxed{\frac{1}{120}}$ .

4-6) The l.c.m of 2, 3, 4, 5, 6 is  $2^2 \cdot 3 \cdot 5 = 60$  so the least such integer is  $\boxed{2^{60}} = (2^{30})^2 = (2^{20})^3 = (2^{15})^4 = (2^{12})^5 = (2^{10})^6$ .