Bergen County Mathematics League

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BCML

Brief Contest Solutions #2

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2-1) Since $\frac{22}{7} = 3.142857\ldots$ and $\pi = 3.14159\ldots$, we see that $|\pi - 22/7| = 0.00126\ldots$ and $|\pi - 3.14| = 0.00159$, 3.14 is not as close an approximation to π as is $\boxed{22/7}$.

2-2) Tetrahedral faces are triangular. A cube face needs at least 2 tetrahedral faces, so at least 12 tetrahedral faces are needed in all. At most 3 faces of a tetrahedron are mutually orthogonal (no 2 faces can be parallel), so at most 3 faces from each tetrahedron can contribute towards these 12. We need ≥ 4 tetrahedrons to provide the cube's faces. The volume of each tetrahedron is 1/6 that of the cube (1/3×face area×1, and face area is 1/2 a face of the cube). Together, 4 such tetrahedra are less than the volume of 1 cube, so we need ≥ 5 tetrahedra. Here's how to get 5 such tetrahedral whose union is the cube: lop of tetrahedron from 4 non-adjacent corners of the cube, and you will leave 1 tetrahedron. More precisely, take the cube as *ABCD-A'B'C'D'*, with *ABCD* horizontal and *A'* directly under *A*, *B'* directly under *B*, and so on. The 5 tetrahedra are *AA'BD*, *CC'BD*, *DD'A'C'*, *BB'A'C'*, and *BDA'C'*.

2-3) If $x<0$, the left side is greater than the right side.
If $x=0$, " " " " " " " " ".
If $x>0$, " " " " " " " " ".
The solutions are $\boxed{\{\text{all real numbers}\}}$.

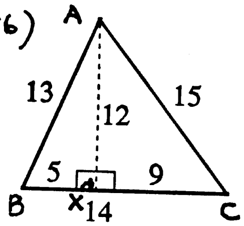
2-4)

	Now	Then	Future
Man	$4x$	$3x$	$5x$
Son	$3x$	$2x$	$4x$

Let $3x$ = the son's current earnings. We are told that $5x+4x = \$117000$, so $x = \$13000$ and the man's current earnings ar $4x = 4(\$13000) = \boxed{\$52000}$.

2-5) As shown in the solution to 1-4), if x_1 is a solution to $Ax^3+Bx^2+Cx+D=0$ then $\frac{1}{x_1}$ is a solution of $Dx^3+Cx^2+Bx+A=0$, where $AD\neq 0$. Therefore, the solutions are $\boxed{\frac{17}{12}, \frac{-11}{23}, \text{ and } \frac{29}{19}}$.

2-6)



$\text{Max}\left(\frac{1}{AX}+\frac{1}{BY}+\frac{1}{CZ}\right) = \text{Min}(AX+BY+CZ)$. The minimum in each case is the altitude from the opposite vertex. Since the area of $\triangle ABC = 84$, as shown, we have $\frac{1}{2}(14)(AX)=84$, $\frac{1}{2}(15)(BY)=84$, and $\frac{1}{2}(13)(CZ)=84$. Solving, $AX=\frac{168}{14}$, $BY=\frac{168}{15}$, and $CZ=\frac{168}{13}$. The max sought is $\frac{14}{168}+\frac{15}{168}+\frac{13}{168}=\boxed{1/4}$.

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